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ferred for the beginner. The change of intellectual climate is less sudden; the introduction of novel conceptions less bewildering; and the powerful methods of the modern synthesis will be better appreciated for this gradual revelation of their freedom and scope. In a somewhat careful reading no error likely to mislead the student has been found except in the statements as to reciprocal polars in 82 *seq.*, which are entirely wrong, and inconsistent with the correct statements in 77–81. The locus of Q is the *inverse* to the locus of P, not the *reciprocal polar*. And the statement of 83, that the locus in the case of a circle is a conic, is not only wrong, but contradicts the correct statement of 82.

The well-selected exercises are carefully assorted and appended to the appropriate sections, instead of being thrown confusedly to the end of the book. To each exercise are added hints for its solution; these would seem to furnish an amount of help to the learner always ample, perhaps in some cases excessive; a few difficulties might have been left unsolved. [*W. M. T.*]

SOLUTIONS OF EXERCISES.

ACKNOWLEDGMENTS.

R. D. Bohannan, 145, 146, 147, 148, 149, 150; W. W. Campbell, 152; G W. Coakley, 151, 152, 156; R. H. Graves, 147, 150, 153, 154; A. Hall, 145, 146, 148, 149, 150, 152; Ellen A. Hayes, 152; J. E. Hendricks, 156; Wm. Hoover, 152; H. A Howe, 156; J. M. Ingalls, 156; Alfred C. Lane, 151, 155, 156. Artemas Martin, 155; J. S. Miller, 147, 148.

145

FIND the equation to the circle through the feet of the normals from (h, k) to the parabola $y^2 = 2px$.

SOLUTION.

Circle $x^2 + y^2 + 2gx + 2fy + c$ cuts parabola $y^2 - 2px$ in points whose ordinates are roots of

$$y^4 + y^2(4p^2 + 4pg) + 8p^2fy + 4p^2c = 0. \qquad (1)$$

The ordinates of the feet of the normal from (h, k) on $y^2 - 2px$ are roots of

$$y^3 + y(2p^2 - hp) - 2p^2k = 0. \qquad (2)$$

Equations (1) and (2) have three common roots if

$$2f = -\tfrac{1}{2}k, \quad 2g = -p - \tfrac{1}{2}h, \quad c = 0.$$

[*R. D. Bohannan, and others*].

146

FIND the condition that the chords AB, CD in an ellipse should meet the transverse axis in points equidistant from the centre, the points A, B, C, D being given by their eccentric anomalies α, β, γ, δ.

SOLUTION.

Chords AB, CD make on the transverse axis intercepts

$$a \cos \tfrac{1}{2}(\alpha - \beta) \sec \tfrac{1}{2}(\alpha + \beta), \quad a \cos \tfrac{1}{2}(\gamma - \delta) \sec \tfrac{1}{2}(\gamma + \delta).$$

These are equal and opposite if

$$\tan \tfrac{1}{2}\alpha \tan \tfrac{1}{2}\beta \tan \tfrac{1}{2}\gamma \tan \tfrac{1}{2}\delta = 1.$$

[*A. Hall, and others.*]

147

THE point P on an ellipse is the projection of Q on the circle on the transverse axis. $P'Q'$ are the points diametrically opposite. Find the area of the parallelogram formed by the four points in terms of the eccentric anomaly of P.

SOLUTION.

Angle $PQQ' = 90° - p$; $PQ = (a - b) \sin p$; $QQ' = 2a$; hence the required area is

$$2a(a - b) \sin p \sin(90° - p) = a(a - b) \sin 2p.$$

148

FIND the area of the parallelogram formed by tangents at the same points.

SOLUTION.

The tangent at P is $ay \sin p + bx \cos p - ab$. This is cut by the diameter $-x + y \tan p$, parallel to the tangent at Q, where

$$y = \frac{ab}{(a - b) \sin p}.$$

The required area is $4ay \sec p$.

149

FROM a point P whose distances from the foci of an ellipse are r, r' tangents are drawn to the curve. Find the angle between them.

SOLUTION.

Let FM, $F'M' = r, r'$ be focal radii to the points of contact. Intersect them in N, N' by circles about F', F with radius $2a$. Then the sides of PFN', $PF'N$ are r, r', $2a$, and angles $F'PN$, FPN' are equal. Therefore MPN' equals FPN' and

$$\cos MPN' = \frac{r^2 + r'^2 - 4a^2}{2rr'}.$$

[Salmon, *Conics*, p. 192.]

150

SHOW that the equation to the pair of tangents from $x'y'$ to $b^2x^2 + a^2y^2 - a^2b^2$ may be written

$$b^2(x - x')^2 + a^2(y - y')^2 = (xy' - x'y)^2.$$

SOLUTION.

Transfer the origin to $x'y'$, and express the condition that the intersections of the ellipse with $y - mx$ shall coincide. This condition is reducible to

$$a^2m^2 + b^2 = (y' - mx')^2.$$

Whence the result follows easily.

151

INTEGRATE the differential

$$\frac{\sin\,\theta + \cos\,\theta}{\sin^4\theta + \cos^4\theta}\,d\theta.$$ [*E. Frisby.*]

SOLUTION.

Substitute $x = \sin\theta - \cos\theta$; we get

$$du = \frac{2dx}{1 + 2x^2 - x^4};$$

whence $$u = \frac{\surd 2}{4\alpha}\log\frac{x+\alpha}{x-\alpha} + \frac{\surd 2}{2\beta}\tan^{-1}\frac{x}{\beta},$$

where $\alpha = \surd[\surd(2+1)]$, $\beta = \surd[\surd(2-1)]$. [*G. W. Coakley, and others.*]

152

IF r and r' be *radii vectores* in a parabola, and $2f$ the difference of the true anomalies, the area of the parabolic sector between the *radii* is

$$\tfrac{1}{3}\surd(rr')\sin f\,.\,[r + r' + \surd(rr')\cos f].$$ [*A. Hall.*]

SOLUTION.

The area of the segment between the chord PP', and the arc of $y^2 - 2p - x$ is

$$\frac{(y' - y)^3}{12p};$$

we have also $x' - x = r' - r$ and $\theta' - \theta = 2f$; and from the geometrical properties of the curve

$$p = 2r\cos^2\tfrac{1}{2}\theta = 2r'\cos^2\tfrac{1}{2}\theta',$$

$$r^2 - 2rr'\cos 2f + r'^2 = (r' - r)^2 + (y' - y)^2.$$

Elimination of p, y, y' gives for the area of the segment

$$\tfrac{1}{3}\surd(rr')\sin f\,.\,[r + r' - 2\surd(rr')\cos f].$$

Add to this the area of the triangle $PFP' = rr'\sin f\cos f$. [*A. Hall.*]

153

THE normals to a parabola at the points P, Q, R pass through the point O and meet the axis at the points P_1, Q_1, R_1; S is the focus and $4a$ is the length of the *latus rectum*; show that

$$PP_1 \,.\, QQ_1 \,.\, RR_1 = 8a^2 \,.\, SO.$$

[*R. H. Graves.*]

SOLUTION.

The length of the normal is $n = \sqrt{(p^2 + y'^2)}$. Eliminate by the aid of this relation y' from the equation to the normal. We get the cubic in n^2

$$(n^2 + p^2 - 2px)^2 (n^2 + p^2) = 4p^4y^2.$$

The product of the roots gives

$$n_1n_2n_3 = 2p^2 \sqrt{[y^2 + (x - \tfrac{1}{2}p)^2]}.$$

154

FIND the locus of the point from which the four normals drawn to an equilateral hyperbola form an harmonic pencil. [*R. H. Graves*]

SOLUTION.

The directions of the four normals from (x,y) to $x^2 - y^2 - a^2$ are found from

$$x^2m^4 - 2xym^3 - (x^2 - y^2 - 4a^2)\, m^2 + 2xym - y^2 = 0.$$

These form an harmonic pencil if

$$8x^2(x^2 - y^2 - 4a^2)y^2 + 2x^2y^2(x^2 - y^2 - 4a^2) = 12x^4y^2 - 12x^2y^4 - \tfrac{2}{9}(x^2 - y^2 - 4a^2)^3$$

That is, if $$(x^2 - y^2 - 4a^2)^3 = 216a^2x^2y^2.$$ [*R. H. Graves.*]

155

A tells the truth *b* times out of *a*; *B*, whose veracity is unknown, joins *A* in affirming the occurrence of an event. What is the probability that the event did occur? [*Artemas Martin.*]

SOLUTION.

If $c = b \div a$ is the probability of any statement by A; x that of any statement by B; the probability that they agree in the truth is cx, in an untruth is $(1 - c)(1 - x)$. The probability sought, if x were known, would be

$$p_x = \frac{cx}{cx + (1 - c)(1 - x)}$$

As x is unknown, and may have any value from 0 to 1, the required probability is

$$p = \int_0^1 p_x dx \div \int_0^1 dx = \frac{b}{2b - a}\left[1 - \frac{a - b}{2b - a} \log \frac{b}{a - b}\right].$$

[*Artemas Martin.*]